SECTION 7.1 AND 7.5: INVERSES AND THE INVERSE CIRCULAR FUNCTIONS

RECALL: Two functions f and g are called **inverses** if:

$$g(f(x)) = x \quad \text{for all } x \text{ in the domain of } f \quad \textbf{AND} \quad f(g(x)) = x \quad \text{for all } x \text{ in the domain of } g$$

Procedurally, the function g 'undoes' the function f and vice versa . In this case, we write $g = f^{-1}$. In symbols:

$$x \xrightarrow{\text{apply } f} f(x) \xrightarrow{\text{apply } f^{-1}} f^{-1}(f(x)) = x$$

Likewise, $f = g^{-1} = (f^{-1})^{-1}$:

$$x \xrightarrow{\text{apply } f^{-1}} f^{-1}(x) \xrightarrow{\text{apply } f} f(f^{-1}(x)) = x$$

If f^{-1} exists, we say the function f is **invertible**.

INVERSE FUNCTION SUMMARY: From College Algebra, we know if f is invertible:

- $f(a) = b \iff f^{-1}(b) = a$.
- the point (a, b) is on the graph of $y = f(x) \iff$ the point (b, a) is on the graph of $y = f^{-1}(x)$.
- the graph of $y = f^{-1}(x)$ can be obtained by reflecting the graph of $y = f(x)$ across the line $y = x$.
- the domain of f is the range of f^{-1} and vice-versa

Not all functions are invertible! Take the classic example: $f(x) = x^2$. Why isn't f invertible? Suppose g is the inverse of f. Since $f(-2) = (-2)^2 = 4$, it must be that $g(4) = -2$. Likewise, since $f(2) = (2)^2 = 4$, we need to have $g(4) = 2$, as well. Since g is a **function**, we can't have $g(4)$ be both -2 **and** 2, so f isn't invertible.[1]

In order for a function f to be invertible, we can't have different inputs go to the same output. Functions which do take different inputs to different outputs are called **one-to-one**. Said differently, for f to be a function, each input can go to only **one** output and for f to have an inverse, each output can come from only **one** input.

We summarize the conditions for invertibility below.

CONDITIONS FOR INVERTIBILITY: The following are equivalent for a function f:

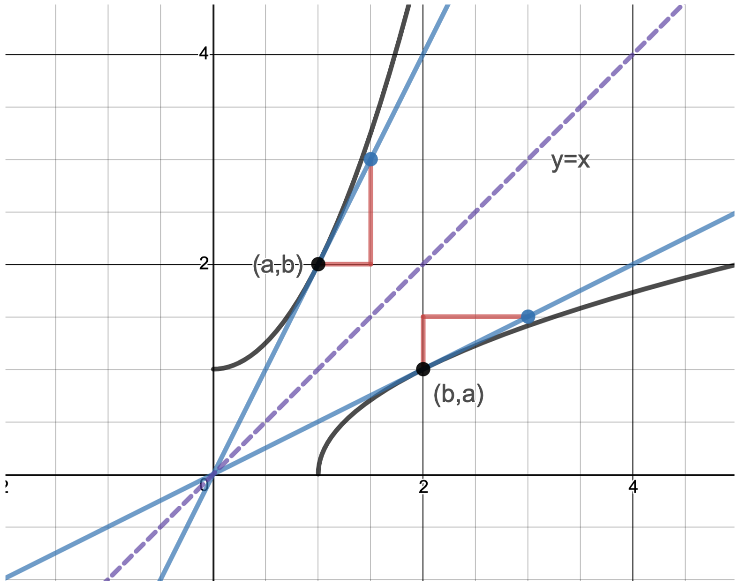
- f is invertible
- $f(x_1) = f(x_2) \iff x_1 = x_2$ (that is, f is **one-to-one**)
- the graph of $y = f(x)$ intersects each Horizontal Line at most once.
  (that is, the graph of f **passes** the so-called **Horizontal Line Test**.)

[1]Note that $g(x) = \sqrt{x}$ is only a **partial** inverse to $f(x) = x^2$. We know that $f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$. However, $g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$. Since $|x| = x$ only if $x \geq 0$, $g(x) = \sqrt{x}$ is only an inverse to $f(x) = x^2$ on the restricted domain $x \geq 0$.

DERIVATIVES OF INVERSES: Let f be an invertible, differentiable function. If $f(a) = b$ and $f'(a) \neq 0$, then

$$\left(f^{-1}\right)'(b) = \frac{1}{f'(a)}$$

In other words, the slope of $y = f^{-1}(x)$ at (b, a) is the reciprocal of the slope of $y = f(x)$ at (a, b).
The proof of this result can best be visualized using a graph as seen below. The key is that when passing from the graph of $y = f(x)$ to the graph of $y = f^{-1}(x)$, the x- and y-coordinates of all the points switch.



EXAMPLE 1: Suppose f is invertible with $f(2) = 5$ and $f'(2) = -3$.

1. Find $\left(f^{-1}\right)'(5)$.

   Using the formula for the derivatives of inverses we get: $\left(f^{-1}\right)'(5) = \dfrac{1}{f'(2)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

2. Write the equation of the tangent line to $y = f^{-1}(x)$ when $x = 5$.

   We know the tangent line formula is: $y = \left(f^{-1}\right)'(5)(x - 5) + f^{-1}(5)$.

   We just determined that $\left(f^{-1}\right)'(5) = -\frac{1}{3}$. Since $f(2) = 5$, we know $f^{-1}(5) = 2$.

   Hence our tangent line is: $y = -\frac{1}{3}(x - 5) + 2$ or, after simplifying, $y = -\frac{1}{3}x + \frac{11}{3}$.

EXAMPLE 2: (VIDEO) Suppose f is invertible with $f(0) = 1$ and $f'(0) = 2$.

1. Find $\left(f^{-1}\right)'(1)$.

   Ans: $\left(f^{-1}\right)'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{2}$

2. Write the equation of the tangent line to $y = f^{-1}(x)$ when $x = 1$.

   Ans: $y = \dfrac{1}{2}x - \dfrac{1}{2}$

If f is an invertible function and $f(a) = b$ then $a = f^{-1}(b)$. Hence we may rewrite:

$$\left(f^{-1}\right)'(b) = \frac{1}{f'(a)} = \frac{1}{f'(f^{-1}(b))}$$

Hence, this gives us the following formula for the derivatives of inverse functions:

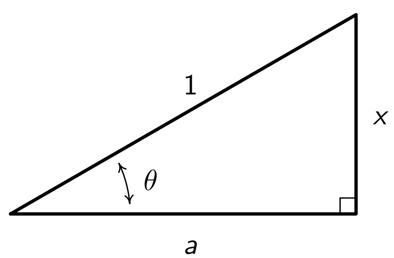
$$D_x[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$$

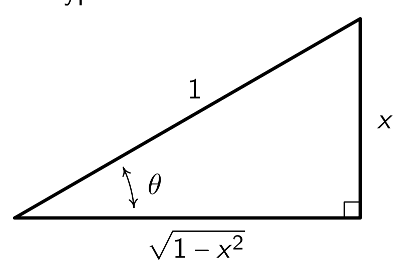
EXAMPLE 3: Recall $f(x) = \sin(x)$ is one-to-one if we restrict the domain to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
We denote the inverse in this case as $f^{-1}(x) = \sin^{-1}(x)$ or $f^{-1}(x) = \arcsin(x)$.
We know for $f(x) = \sin(x)$, $f'(x) = \cos(x)$ so we can use the formula for the derivative of inverses to find:

$$D_x[\arcsin(x)] = \frac{1}{f'(\arcsin(x))} = \frac{1}{\cos(\arcsin(x))}$$

To simplify $\cos(\arcsin(x))$ we let $\theta = \arcsin(x)$. Our problem then boils down to finding an expression for $\cos(\theta)$ where θ is an angle $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ with $\sin(\theta) = x$. One way to solve the problem is to use a right triangle diagram.

We imagine θ as an acute angle and think of $\sin(\theta) = x = \dfrac{x}{1} = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$.





To find an expression for $\cos(\theta)$, we use the Pythagorean Theorem to find an expression for the adjacent side of the triangle. We have $a^2 + x^2 = 1^2$ so $a^2 = 1 - x^2$ or $a = \sqrt{1-x^2}$. Hence,

$$\cos(\arcsin(x)) = \cos(\theta) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

There is a bit more to do here, however, since not all angles θ under consideration are acute. We leave those considerations to the reader. (Hint: if $-\frac{\pi}{2} \le \theta \le 0$, then consider a triangle above but in Quandrant IV . . .)
Alternatively, we can use the identity: $\cos^2(\theta) + \sin^2(\theta) = 1$ to get $\cos(\theta) = \pm\sqrt{1 - \sin^2(\theta)} = \pm\sqrt{1-x^2}$.

Since $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, $\cos(\theta) \ge 0$ so we get $\cos(\arcsin(x)) = \cos(\theta) = \sqrt{1-x^2}$ in this case as well.

Using the identity method, we can consider all angles θ in the range $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ without having special cases.

Hence, using either method, $\cos(\arcsin(x)) = \sqrt{1-x^2}$, so $D_x[\arcsin(x)] = \dfrac{1}{\sqrt{1-x^2}}$.

Alternatively, we can use implicit differentiation: $y = \sin^{-1}(x)$ means $\sin(y) = x$.

Taking the derivative of both sides with respect to 'x' gives $\cos(y)\, y' = 1$. Hence,

$$y' = \frac{1}{\cos(y)} = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}} \checkmark$$

Using the above sort of arguments, we get:

DERIVATIVES OF THE INVERSE CIRCULAR FUNCTIONS:

- $D_x[\sin^{-1}(x)] = D_x[\arcsin(x)] = \dfrac{1}{\sqrt{1-x^2}}$
- $D_x[\cos^{-1}(x)] = D_x[\arccos(x)] = -\dfrac{1}{\sqrt{1-x^2}}$
- $D_x[\sec^{-1}(x)] = D_x[\text{arcsec}(x)] = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $D_x[\csc^{-1}(x)] = D_x[\text{arccsc}(x)] = -\dfrac{1}{|x|\sqrt{x^2-1}}$
- $D_x[\tan^{-1}(x)] = D_x[\arctan(x)] = \dfrac{1}{1+x^2}$
- $D_x[\cot^{-1}(x)] = D_x[\text{arccot}(x)] = -\dfrac{1}{1+x^2}$

We combine these new formulas with our repertoire of other derivative formulas in the following examples.

EXAMPLE 4: Find the indicated derivatives.

1. For $f(x) = \sin^{-1}(2x)$ find $f'(x)$.
   Using the chain rule, we get:

$$\begin{aligned} D_x[\sin^{-1}(2x)] &= \frac{1}{\sqrt{1-(2x)^2}} \cdot D_x[2x] \\ &= \frac{2}{\sqrt{1-4x^2}} \end{aligned}$$

2. If $F(x) = \tan^{-1}(x^2)$, find $F''(x)$.
   Again, the chain rule gives:

$$\begin{aligned} F'(x) &= D_x[\tan^{-1}\left(x^2\right)] \\ &= \frac{1}{1+(x^2)^2} \cdot D_x[x^2] \\ &= \frac{2x}{1+x^4} \end{aligned}$$

   Now, using the quotient rule, we get:

$$\begin{aligned} F''(x) &= D_x\left[\frac{2x}{1+x^4}\right] \\ &= \frac{\left(1+x^4\right) D_x[2x] - (2x)\, D_x\left[1+x^4\right]}{(1+x^4)^2} \\ &= \frac{\left(1+x^4\right)(2) - (2x)\left(4x^3\right)}{(1+x^4)^2} \\ &= \frac{2+2x^4-8x^4}{(1+x^4)^2} \\ F''(x) &= \frac{2-6x^4}{(1+x^4)^2} \end{aligned}$$

EXAMPLE 5: Find the equation of the tangent line to $y = \sec^{-1}(5x)$ at $x = \frac{\sqrt{2}}{5}$.

First we find the derivative, $\frac{dy}{dx}$. Using the chain rule, we get:

$$\begin{aligned}
\frac{dy}{dx} &= D_x[\sec^{-1}(5x)] \\
&= \frac{1}{|5x|\sqrt{(5x)^2-1}} \cdot D_x[5x] \\
&= \frac{5}{|5x|\sqrt{25x^2-1}} \\
&= \frac{5}{|5|\,|x|\sqrt{25x^2-1}} \\
&= \frac{\cancel{5}}{\cancel{5}|x|\sqrt{25x^2-1}} \\
\frac{dy}{dx} &= \frac{1}{|x|\sqrt{25x^2-1}}
\end{aligned}$$

Hence, the **slope** of the tangent line at $x = \frac{\sqrt{2}}{5}$ is

$$\left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{2}}{5}} = \frac{1}{\left|\frac{\sqrt{2}}{5}\right|\sqrt{25\left(\frac{\sqrt{2}}{5}\right)^2-1}} = \frac{5}{\sqrt{2}\sqrt{2-1}} = \frac{5}{\sqrt{2}}$$

Since $\sec^{-1}\left(5\frac{\sqrt{2}}{5}\right) = \sec^{-1}(\sqrt{2}) = \frac{\pi}{4}$, we get our tangent line at $x = \frac{\sqrt{2}}{5}$ to be:

$$y = \frac{5}{\sqrt{2}}\left(x - \frac{\sqrt{2}}{5}\right) + \frac{\pi}{4} \quad \text{or} \quad y = \frac{5}{\sqrt{2}}x - 1 + \frac{\pi}{4}$$

We invite the reader to check this using a graphing utility.

EXAMPLE 6: (VIDEO) Find the indicated derivative.

1. For $f(x) = x\arcsin(x) + \sqrt{1-x^2}$ find $f'(x)$.

   Ans: $f'(x) = \arcsin(x)$.

2. Find $D_x[\tan^{-1}(\sqrt{x})]$

   Ans: $D_x[\tan^{-1}(\sqrt{x})] = \dfrac{1}{2x^{3/2}+2x^{1/2}}$

3. Find $\frac{d}{dx}\left[\sec^{-1}\left(x^3\right)\right]$

   Ans: $\dfrac{d}{dx}\left[\sec^{-1}\left(x^3\right)\right] = \dfrac{3}{|x|\sqrt{x^6-1}}$

INTEGRATION FORMULAS INVOLVING INVERSE CIRCULAR FUNCTIONS:

- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C = \sin^{-1}\left(\frac{x}{a}\right) + C$
- $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\text{arcsec}\left(\frac{|x|}{a}\right) + C = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C$

EXAMPLE 7: Find the following antiderivatives. Check you answer by taking the derivative.

1. $\displaystyle\int \frac{1}{x^2+25}\,dx$

$$\int \frac{1}{x^2+25}\,dx = \int \frac{1}{x^2+(5)^2}\,dx = \frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$$

We find after the chain rule and some algebra that $D_x\left[\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C\right] = \ldots = \frac{1}{x^2+25}$ ✓.

2. $\displaystyle\int \frac{1}{\sqrt{3-x^2}}\,dx$

$$\int \frac{1}{\sqrt{3-x^2}}\,dx = \int \frac{1}{\sqrt{\left(\sqrt{3}\right)^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

We find after the chain rule and some algebra that $D_x\left[\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + C\right] = \ldots = \frac{1}{\sqrt{3-x^2}}$ ✓.

3. $\displaystyle\int \frac{1}{x\sqrt{4x^2-36}}\,dx$

$$\int \frac{1}{x\sqrt{4x^2-36}}\,dx = \int \frac{1}{x\sqrt{4\left(x^2-9\right)}}\,dx = \int \frac{1}{2x\sqrt{x^2-9}}\,dx = \frac{1}{2}\int \frac{1}{x\sqrt{x^2-9}}\,dx = \frac{1}{6}\sec^{-1}\left(\frac{|x|}{3}\right) + C$$

Once again, after the chain rule and some algebra, we find that $D_x\left[\frac{1}{6}\sec^{-1}\left(\frac{|x|}{3}\right) + C\right] = \ldots = \frac{1}{x\sqrt{4x^2-36}}$ ✓.

EXAMPLE 8: (VIDEO) Find the following integrals. Check your answer using differentiation.

1. $\displaystyle\int \frac{1}{x^2+16}\,dx$

   Ans: $\displaystyle\int \frac{1}{x^2+16}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$

2. $\displaystyle\int \frac{1}{\sqrt{5-x^2}}\,dx$

   Ans: $\displaystyle\int \frac{1}{\sqrt{5-x^2}}\,dx = \sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

3. $\displaystyle\int \frac{1}{x\sqrt{9x^2-36}}\,dx$

   Ans: $\displaystyle\int \frac{1}{x\sqrt{9x^2-36}}\,dx = \frac{1}{6}\sec^{-1}\left(\frac{|x|}{2}\right) + C$

EXAMPLE 9: (VIDEO) Find the area under the curve $y = \dfrac{1}{x^2+4}$ from $x = 0$ to $x = 2$.

Ans: Area $= \displaystyle\int_0^2 \frac{1}{x^2+4}\,dx = \ldots = \frac{\pi}{8}$ units^2

EXAMPLE 10: (VIDEO) Find the following integrals. Check your answer using differentiation.

1. $\displaystyle\int \frac{1}{\sqrt{9-25x^2}}\,dx$

   Ans: $\displaystyle\int \frac{1}{\sqrt{9-25x^2}}\,dx = \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$

2. $\displaystyle\int \frac{x}{x^4+1}\,dx$

   HINT: $x^4+1 = \left(x^2\right)^2 + 1\ldots$

   Ans: $\displaystyle\int \frac{x}{x^4+1}\,dx = \frac{1}{2}\tan^{-1}\left(x^2\right) + C$

3. $\displaystyle\int \frac{1}{x^2+2x+5}\,dx$

   HINT: Complete the Square: $x^2+2x+5 = \left(x^2+2x+1\right)+4 = (x+1)^2+4\ldots$

   Ans: $\displaystyle\int \frac{1}{x^2+2x+5}\,dx = \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$

HOMEWORK:

Section 7.1: Algebra Review: 5, 23 - 37 odd. Calculus problems: 39 - 55 odd, 61 - 67 odd.

Section 7.5: Trigonometry Review: 17 - 47 odd, 93, 95, 107, 109.

Calculus problems: 49, 51, 55 - 63 odd, 71, 73, 77 - 83 odd, 101-102*, 103*

APPENDIX: THE POWER RULE FOR RATIONAL EXPONENTS

When we introduced the power rule, $D_x\left[x^k\right] = k\,x^{k-1}$, we noted that the formula works for all real numbers k. At the time, we only had the tools to verify the power rule for integers (that is $k = 0, \pm 1, \pm 2, \pm 3, \ldots$).
With the arrival of the derivative formula for inverse functions:

$$D_x[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))},$$

we can justify the power rule for root functions.

Recall that $g(x) = \sqrt[n]{x}$ can be defined as the inverse for $f(x) = x^n$ with the caveat that if n is even, the domain of f is restricted to $x \geq 0$. Using the derivative formula for inverses, we get

$$\begin{aligned}
D_x[\sqrt[n]{x}] = D_x\left[x^{\frac{1}{n}}\right] &= \frac{1}{f'\left(x^{\frac{1}{n}}\right)} \\
&= \frac{1}{n\left(x^{\frac{1}{n}}\right)^{n-1}} && f(x) = x^n \text{ so } f'(x) = n\,x^{n-1} \\
&= \frac{1}{n\,x^{\frac{1}{n}(n-1)}} && \text{Law of Exponents: } (a^p)^q = a^{pq} \\
&= \frac{1}{n\,x^{1-\frac{1}{n}}} \\
&= \frac{1}{n}\,x^{-\left(1-\frac{1}{n}\right)} && \text{Properties of Exponents: } \frac{1}{a^p} = a^{-p} \\
&= \frac{1}{n}\,x^{-1+\frac{1}{n}} \\
D_x[\sqrt[n]{x}] = D_x\left[x^{\frac{1}{n}}\right] &= \frac{1}{n}\,x^{\frac{1}{n}-1} \checkmark
\end{aligned}$$

With the power rule for root functions established, we can extend the power rule to rational number exponents by noting that: $f(x) = x^{\frac{p}{q}} = (x^p)^{\frac{1}{q}}$ and using the chain rule:

$$D_x\left[x^{\frac{p}{q}}\right] = D_x\left[(x^p)^{\frac{1}{q}}\right] = \frac{1}{q}\,(x^p)^{\frac{1}{q}-1}\,D_x\left[x^p\right] = \frac{1}{q}\,x^{p\left(\frac{1}{q}-1\right)}p\,x^{p-1} = \frac{p}{q}\,x^{\frac{p}{q}-p}\,x^{p-1} = \frac{p}{q}\,x^{\frac{p}{q}-p+p-1} = \frac{p}{q}\,x^{\frac{p}{q}-1} \checkmark$$

MATH 2500: INVERSE CIRCULAR FUNCTIONS REVIEW

- $\sin^{-1}(x) = \arcsin(x)$ is an angle between $-\pi/2$ and $\pi/2$ (Quadrant I or –IV) whose sine is x.
- $\csc^{-1}(x) = \text{arccsc}(x)$ is an angle between $-\pi/2$ and $\pi/2$ (Quadrant I or –IV) whose cosecant is x.
- $\tan^{-1}(x) = \arctan(x)$ is an angle between $-\pi/2$ and $\pi/2$ (Quadrant I or –IV) whose tangent is x.
- $\cos^{-1}(x) = \arccos(x)$ is an angle between 0 and π (Quadrant I or II) whose cosine is x.
- $\sec^{-1}(x) = \text{arcsec}(x)$ is an angle between 0 and π (Quadrant I or II) whose secant is x.
- $\cot^{-1}(x) = \text{arccot}(x)$ is an angle between 0 and π (Quadrant I or II) whose cotangent is x.

1. **'Walking':** Find the **exact** values of the expressions below without using a calculator.
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$
   (b) $\cot^{-1}(-1)$
   (c) $\arccos(0)$
   (d) $\tan^{-1}(\sqrt{3})$
   (e) $\sec^{-1}(-2)$
   (f) $\text{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$
   (g) $\sin(\arcsin(-1))$
   (h) $\cos(\cos^{-1}(.2))$
   (i) $\arctan(\tan(5\pi/4))$
   (j) $\csc^{-1}(\csc\left(\frac{\pi}{3}\right))$
   (k) (TRICK!) $\text{arcsec}(\sec(\pi/2))$ Explain why this is a trick question.

2. **'Jogging':** Find the **exact** values of the expressions below without using a calculator.
   (a) $\sec(\arctan(-1))$
   (b) $\tan(\arccos(-1/2))$
   (c) $\cos(\cot^{-1}(-3/4))$
   (d) $\csc(\arcsin(.3))$
   (e) $\cot(\sec^{-1}(-3))$
   (f) Find an algebraic expression for $\tan(\arcsin(2x))$.

3. **'Running':**
   (a) Explain why $\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$ and $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$
   (b) Find a way to graph $y = \csc^{-1}(x)$ and $y = \sec^{-1}(x)$ on your calculator. (HINT: Use the result from the previous problem!)
   (c) Find $\cot^{-1}(-5)$ to four decimal places.

4. **Tournament of Champions!**
   (a) Find an algebraic expression for $\tan(\text{arcsec}(x))$.
   (b) Come up with a way to use your calculator to graph $y = \cot^{-1}(x)$.

Answers.

1. (a) $\pi/6$
   (b) $3\pi/4$
   (c) $\pi/2$
   (d) $\pi/3$
   (e) $2\pi/3$
   (f) $-\pi/3$
   (g) -1
   (h) $.2$
   (i) $\pi/4$
   (j) $\pi/3$
   (k) $\sec(\pi/2)$ is undefined!
2. (a) $\sqrt{2}$
   (b) $-\sqrt{3}$
   (c) $-3/5$
   (d) $10/3$
   (e) $-\sqrt{2}/4$
   (f) $\dfrac{2x}{\sqrt{1-4x^2}}$
3. (a) Draw a picture or use the definition of the functions in the appropriate quadrants.
   (b) Graph $y = \sin^{-1}(1/x)$ instead of $y = \csc^{-1}(x)$ and graph $y = \cos^{-1}(1/x)$ instead of $y = \sec^{-1}(x)$.
   (c) $\cot^{-1}(-5) = \pi - \tan^{-1}(1/5) \approx 2.9442$. (Can you see why using the Unit Circle?)
4. (a) $\tan(\text{arcsec}(x)) = \sqrt{x^2-1}$, if $x > 0$ and $\tan(\text{arcsec}(x)) = -\sqrt{x^2-1}$ if $x < 0$.
   (b) You may want to break it up into pieces...